

VHEMBE TEACHERS' WORKSHOP 2022 Sophie/Molf/Happy & Frank

Work, Energy & Power



Overview

- Newton's Laws are utilized to analyze the motion of objects
- ➢ Motion is investigated from the perspective of work and energy.
- ➤The concept of force, is related to the concepts of work and energy.

Learners' Performance in each question





DBE REPORT 2019

Common Errors and Misconceptions

- □ Some candidates could not properly define conservative force, while some omitted keywords in their definition, e.g., omitting the word 'work' and using 'force' instead of 'work'.
- □ Many candidates could not differentiate between a conservative force and a non-conservative force acting on the object.
- □ Many candidates wrote a direction for work done on an object, which is a misconception since work done is a scalar quantity.



Suggestions for Improvement!!!



Work Done by a Constant Force



W = Fs $1 \text{ N} \cdot \text{m} = 1 \text{ joule } (J)$



- 1.5 The base SI unit of the physical quantity 'work' is ...
 - A kg·m·s⁻¹
 - B kg·m²·s²
 - C kg·m²·s⁻²
 - D kg·m·s⁻²





 $\cos 180^\circ = -1$



Dependence of W on the direction of F relative to the displacement







(a)

$$W = (F\cos 0)s = Fs$$



$$W = (F\cos 180)s = -Fs$$



(c)



The scientific way of Interpreting 'work'

- One important difference is that *W* (the "physics" work) can be negative.
- An example with W < 0 because the force and displacement are in opposite directions. As a result, the final speed of the object is less than its initial speed.
- In general, we can say that if W > 0, an object will "speed up"; if W < 0, it will "slow down."
- Furthermore, your intuition should suggest that this energy can change when a force acts on the object.
- We will find that situations in which the force *increases* the energy of the object correspond to a positive value of *W*. Conversely, it is possible for a force to *reduce* the energy of an object (as with friction);

in such cases, the work done on the object is negative.



Steps involved in problem-solving:

- Identify the force(s) acting on the object.
- Draw a free body or force diagram and identify the force(s) that do positive and negative work. Copy the formula ($W = F\Delta x \cos \theta$) from the datasheet.
- \bullet The angle θ is between the force and displacement of the object.
- Ensure that all quantities substituted in the formula are in SI units, otherwise carry out the necessary conversions.





Example Pulling a Suitcase-on-Wheels^(b) Find the work done if the force is 45.0-N, the angle is 50.0 degrees and the displacement is 75.0 m.

$$W = (F \cos \theta)s = [(45.0 \text{ N})\cos 50.0^{\circ}](75.0 \text{ m})$$

 $= 2170 \, \text{J}$



Example

In a gravity escape system (GES), an enclosed lifeboat on a large ship is deployed by letting it slide down a ramp and then continue in free fall to the water below. mg Suppose a 4970 kg lifeboat $h = 2.50 \,\mathrm{m}$ slides a distance of 5.00 m on a ramp, dropping through a vertical height of 2.50 m. How much work does gravity do on the boat?



• Guiding Solution

Analysis:

From the sketch, the force of gravity $m\mathbf{g}$ and the displacement \mathbf{d} are at an angle θ relative to one another when placed tail-to-tail, and that θ is also the angle the ramp makes with the vertical. In addition, it can be noted that the vertical height of the ramp is h = 2.50 m, and the length of the ramp is d = 5.00 m.

The work done on the lifeboat by gravity is $W = Fd \cos \theta$, where

F = mg, d = 5.00 m, and θ is the angle between mg and d. Note that θ is not given in the problem statement, but from the right-angled triangle that forms the ramp we see that

$$\cos\theta = \frac{h}{d}$$



First, find the component of $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ in the direction of motion: $F \cos \theta = (mg) \left(\frac{h}{d}\right)$

$$= (4970 \text{ kg})(9.81 \text{ m/s}^2) \left(\frac{2.50 \text{ m}}{5.00 \text{ m}}\right) = 24,400 \text{ N}$$

Multiply by distance to find the work:

$$W = (F \cos \theta)d = (24,400 \text{ N})(5.00 \text{ m}) = 122,000 \text{ J}$$

Alternatively, cancel *d* algebraically before substituting numerical values:

$$W = Fd \cos \theta = (mg)(d) \left(\frac{h}{d}\right)$$

= mgh = (4970 kg)(9.81 m/s²)(2.50 m) = 122,000 J



Example Accelerating a Crate

The truck is accelerating at a rate of $\pm 1.50 \text{ m/s}^2$. The mass of the crate is 120 kg and it does not slip. The magnitude of the displacement is 65 m.

What is the total work done on the crate by all of the forces acting on it?







The angle between the displacement and the normal force is 90°.

The angle between the displacement and the weight is also 90°.

$$W = F \cos 90^\circ = 0$$







The angle between the displacement and the friction force is 0 degrees.

$$f_s = ma = (120 \text{ kg})(1.5 \text{ m/s}^2) = 180 \text{ N}$$

 $W = [(180N)\cos 0](65m) = 1.2 \times 10^4 J$







Activity about Energy

- *Def.:* Energy is the capacity to do work/the ability to cause a change in matter.
- Drop a ball from an increasing height into a tray of wet sand. Start at a height of 25 cm and repeat from heights of 50 cm, 1 m, and 1.5 m. Ask learners the following questions:
- Which needs the most energy to make it a deep crater or a shallow one? Why?

Students should say the deep crater and may say that digging a deep hole takes more work than digging a shallow one.

- \succ What provided the energy in this case?
- The ball. The ball's energy is transferred to the sand.



Is the ball doing work on the sand, then? Explain.
Yes, because it is making the sand move.

What does the crater depth tell you about the energy supplied by the ball when dropped from an increasing height?

The ball supplies more energy to the sand when it has been dropped from a greater height.

So, which ball had the most energy as it hit the sand?

The one that made the deepest crater.



So, which ball must have had the most energy before it was dropped?

The one that was dropped from the greatest height.

So, what can you say about the energy of the ball when it has landed in the sand?

The energy has been transferred to the sand.

To recap, explain the ball's energy from when it is raised, then as it drops, then as it lands: what can you say about the amount of energy it has?

The ball gains more (potential) energy the higher it is lifted. As it falls, the ball gets faster – so we can say it has more kinetic energy the faster it falls, but it doesn't have more energy overall (because it has less potential energy). When the ball has landed in the sand, it has lost all the energy given to it by lifting it up; it must have lost all the kinetic energy it had just before it hit the sand because we can see it has stopped.

Where did all that energy go?

It was transferred to the sand to move it. Or it was used to do work on the sand.



The Work-Energy Theorem and Kinetic Energy

Consider a constant net external force acting on an object.

The object is displaced a distance *s*, in the same direction as the net force.

$$\sum F$$

S
The work is simply $W = (\sum F)s = (ma)s$



The Work-Energy Theorem and Kinetic Energy

DEFINITION OF KINETIC ENERGY

The kinetic energy KE of an object with mass m and speed v is given by

$$\mathrm{KE} = \frac{1}{2}mv^2$$

Kinetic energy quantifies the amount of work the object can do because of its motion



The Work-Energy Theorem and Kinetic Energy



THE WORK-ENERGY THEOREM

When a net external force does work on and object, the kinetic energy of the object changes according to

$$W = KE_{f} - KE_{o} = \frac{1}{2}mv_{f}^{2} - \frac{1}{2}mv_{o}^{2}$$



The Work-Energy Theorem and Kinetic Energy

Example Deep Space 1

The mass of the space probe is 474-kg and its initial velocities 275 m/s. If the 56.0-mN force acts on the probe throug displacement of $2.42 \times 10^9 \text{m}$, what is its final speed?





The Work-Energy Theorem and Kinetic Energy

$$W = \frac{1}{2} m v_{f}^{2} - \frac{1}{2} m v_{o}^{2}$$

$$W = \left[(\sum F) \cos \theta \right] s$$

$$\vec{F} = \vec{v}_{o} \quad \vec{F} = \vec{v}_{f} \quad \vec{v}_{f}$$



The Work-Energy Theorem and Kinetic Energy

$$\left[\left(\sum F\right)\cos\theta\right]s = \frac{1}{2}mv_{\rm f}^2 - \frac{1}{2}mv_o^2$$





The Work-Energy Theorem and Kinetic Energy



In this case the net force is

$$\sum F = mg \sin 25^\circ - f_k$$



The change in gravitational potential energy when an object moves up or down is the negative of the work done by gravity:

$$\Delta U_{grav} = -W_{grav}$$

If the gravitational field is uniform, the work done by gravity is

$$W_{grav} = -mg\Delta y$$

where the y -axis points up. Therefore, $\Delta U_{qrav} = mg\Delta y$





(a) When the stone moves up, the gravitational potential energy increases.



(b) When the stone moves down, the gravitational potential energy decreases.



• Significance of the negative sign:

When the stone moves up, Δy is positive. The gravitational force and the displacement of the stone are in opposite directions, so the work done by gravity is negative, gravity is taking away kinetic energy and adding it to its stored potential energy, so the potential energy increases.

If the stone moves down, Δy is negative. The work done by gravity is positive; gravity is giving back kinetic energy by depleting its storage of potential energy, so the potential energy decreases



$$W = (F\cos\theta)s$$

$$W_{\text{gravity}} = mg(h_o - h_f)$$









• Consider the illustration of a man lifting a box below and say whether he is demonstrating work.



- Which/who is doing work in the illustration?
- What is the direction of the force exerted by the man on the box?
- What is the direction of the motion of the box?



Example A Gymnast on a Trampoline

The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



$$W = \frac{1}{2} m v_{f}^{2} - \frac{1}{2} m v_{o}^{2}$$

$$W_{\text{gravity}} = mg(h_{o} - h_{f})$$

$$M_{gravity} = mg(h_{o} - h_{f})$$

$$M_{o} = \sqrt{-2g(h_{o} - h_{f})}$$

$$v_o = \sqrt{-2(9.80 \text{ m/s}^2)(1.20 \text{ m} - 4.80 \text{ m})} = 8.40 \text{ m/s}$$



$$W_{\text{gravity}} = mgh_o - mgh_f$$

DEFINITION:

The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level:

$$PE = mgh$$

 $1 \text{ N} \cdot \text{m} = 1 \text{ joule } (J)$



DEFINITION OF A CONSERVATIVE FORCE

Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

Version 2 A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.



Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final position

$$W_{\text{gravity}} = mg(h_o - h_f)$$





Version 2 A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.





An example of a nonconservative force is the kinetic frictional force.

$$W = (F\cos\theta)s = f_k\cos 180^\circ s = -f_ks$$

The work done by the kinetic frictional force is always negative. Thus, it is impossible for the work it does on an object that moves around a closed path to be zero.

The concept of potential energy is not defined for a nonconservative force.

In normal situations both conservative and nonconservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W = W_c + W_{nc}$$
$$W = KE_f - KE_o = \Delta KE$$
$$W_c = W_{gravity} = mgh_o - mgh_f = PE_o - PE_f = -\Delta PE$$

The total work done on an object can always be written as the sum of the work done by conservative forces (W_c) plus the work done by nonconservative forces (W_{nc}) . Since the total work is equal to the change in the object's kinetic energy

$$W = W_c + W_{nc}$$

$$/ \qquad /$$

$$\Delta KE = -\Delta PE + W_{nc}$$

THE WORK-ENERGY THEOREM

$W_{nc} = \Delta KE + \Delta PE$

It is very important to know that non-conservative forces do not imply that total energy is not conserved. The total energy is always conserved. Non-conservative forces mean that mechanical energy isn't conserved in a particular system which implies that the energy has been transferred in a process that isn't reversible.

$$W_{nc} = \Delta KE + \Delta PE = (KE_{f} - KE_{o}) + (PE_{f} - PE_{o})$$

$$W_{nc} = \left(\mathrm{KE}_{\mathrm{f}} + \mathrm{PE}_{\mathrm{f}}\right) + \left(\mathrm{KE}_{\mathrm{o}} + \mathrm{PE}_{\mathrm{o}}\right)$$

$$W_{nc} = E_{f} - E_{o}$$

If the net work on an object by nonconservative forces is zero, then its energy does not change:

$$E_f = E_o$$

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

The total mechanical energy (E = KE + PE) of an object remains constant as the object moves, provided that the net work done by external nonconservative forces is zero.



The Conservation of Mechanical Energy Example A Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the cycle's speed when it strikes the ground on the other side.





$$E_{f} = E_{o}$$

$$mgh_{f} + \frac{1}{2}mv_{f}^{2} = mgh_{o} + \frac{1}{2}mv_{o}^{2}$$
$$gh_{f} + \frac{1}{2}v_{f}^{2} = gh_{o} + \frac{1}{2}v_{o}^{2}$$



$$gh_f + \frac{1}{2}v_f^2 = gh_o + \frac{1}{2}v_o^2$$

$$v_f = \sqrt{2g(h_o - h_f) + v_o^2}$$

$$v_f = \sqrt{2(9.8 \,\mathrm{m/s^2})(35.0 \,\mathrm{m}) + (38.0 \,\mathrm{m/s})^2} = 46.2 \,\mathrm{m/s}$$

Nonconservative Forces and the Work-Energy Theorem



$$W_{nc} = E_f - E_o$$

$$W_{nc} = \left(mgh_f + \frac{1}{2}mv_f^2 \right) - \left(mgh_o + \frac{1}{2}mv_o^2 \right)$$



Nonconservative Forces and the Work-Energy Theorem

Example Fireworks

If the nonconservative force generated by the burning propellant does 425 J of work, what is the final speed of the rocket. Ignore air resistance.

$$W_{nc} = \left(mgh_f + \frac{1}{2}mv_f^2\right) - \left(mgh_o + \frac{1}{2}mv_o^2\right)$$



Nonconservative Forces and the Work-Energy Theorem

$$W_{nc} = mgh_{f} - mgh_{o} + \frac{1}{2}mv_{f}^{2} - \frac{1}{2}mv_{o}^{2}$$

$$W_{nc} = mg(h_{f} - h_{o}) + \frac{1}{2}mv_{f}^{2}$$

$$425 \text{ J} = (0.20 \text{ kg})(9.80 \text{ m/s}^{2})(29.0 \text{ m})$$

$$+ \frac{1}{2}(0.20 \text{ kg})v_{f}^{2}$$

$$v_{f} = 61 \text{ m/s}$$



A compound bow in archery allows the user to hold the bowstring at full draw with considerably less force than the maximum force exerted by the string. The draw force as a function of the string position x for a particular compound bow is shown in aside.

(a) How much work does the archer do on

the bow in order to draw the string from

x = 0 to x = 0.60 m?

(b) If all of this work becomes the kinetic energy of a 0.065 kg arrow, what is the speed of the arrow?





G Guiding Solution

Question Analysis:

- The work done by the archer on the bow is the area under the force versus position graph.
- Determine area under the graph by summing the areas of the triangle from x = 0 to x = 0.20 m, the rectangle from x = 0.20 to x = 0.50 m, and the trapezoid from x = 0.50 to x = 0.60 m.
- Then use the resulting work value together with the work-energy theorem to determine the arrow's kinetic energy and speed.



(a) Step 1: Find the area of the triangle from x = 0 to x = 0.20 m: $W_1 = \frac{1}{2}(base)(height) = \frac{1}{2}(0.20 \text{ m})(350 \text{ N}) = 35 \text{ J}$ Step 2: Find the area of a rectangle from x = 0.20 to x = 0.50 m: $W_2 = (350 \text{ N})(0.50 \text{ m} - 0.20 \text{ m}) = 105 \text{ J}$ Step 3: Find the area of a trapezium from x = 0.50 to x = 0.60 m: $W_3 = \frac{1}{2}(sum \ of \ the length \ of \ the \ parallel \ sides)(height)$ 1

$$=\frac{1}{2}(350 \text{ N}+100 \text{ N})(0.10 \text{ m}) = 22.5 \text{ J}$$

Step 4: Total work done:

$$W = W_1 + W_2 + W_3$$

 $W = 35 \text{ J} + 105 \text{ J} + 22.5 \text{ J} = 163 \text{ J}$



(b) Now, we use the work-energy theorem to determine the final speed.

$$W_{Total} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2$$
$$W_{Total} = \frac{1}{2} m v_f^2 - 0$$
$$v_f = \sqrt{\frac{2W_{Total}}{m}} = \sqrt{\frac{2(163 \text{ J})}{0.065 \text{ kg}}}$$
$$v_f = 71 \text{ m/s}$$

Power

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done OR energy is expended, and it is obtained by dividing the work by the time required to perform the work.

$$\overline{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

joule
$$/s = watt (W)$$



$$\overline{P} = \frac{\text{Change in energy}}{\text{Time}}$$

1 horsepower = 550 foot \cdot pounds/second = 745.7 watts

$$\overline{P} = F\overline{v}$$



• Example 1

A block of 200 kg is pulled along the floor at a constant speed by an electric motor. The coefficient of friction between the block and the floor is 0,2.

1.1 Calculate the frictional force experienced by the block.

1.2 Calculate the power the motor must deliver if the block is to move at a constant speed of 6 m/s.

- 1.3 How much work is done by the motor in 60 s?
- 1.4 What is the net work done on the block in the 60 s?



1.1 N=mg,
$$F_f = \mu$$
mg
 $F_f = (0,2)(200 \text{ kg})9.8 \text{ m/s}^2$
 $F_f = 392 \text{ N}$
1.2 $P = Fv = (392 \text{ N})(6 \text{ m/s})$
 $P = 2 352 \text{ W}$
1.3 $P = \frac{W}{\Delta t} \Longrightarrow 2 352 \text{ W} = \frac{W}{60 \text{ s}}$
 $W = 141 120 \text{ J}$
1.4 $W_{net} = \Delta KE = 0$
Since velocity is constant.

Other Forms of Energy and the Conservation of Energy

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but can only be converted from one form to another.



An object of mass 1,8 kg slides down a rough curved track and passes point **A**, which is 1,5 m above the ground, at a speed of 0,95 m·s-1. The object reaches point **B** at the bottom of the track at a speed of 4 m·s-1.





- 1.1 Define the term *conservative force*. (2)
- 1.2 Name the conservative force acting on the object. (1)

1.3 Is mechanical energy conserved as the object slides from point A to point B? Choose from YES or NO. Give a reason for the answer. (2)

1.4 Calculate the gravitational potential energy of the object when it was at point A. (3)

1.5 Using energy principles, calculate the work done by friction on the object as it slides from point A to point B. (4)

Surface BC in the diagram above is frictionless.

5.6 What is the value of the net work done on the object as it slides from point B to point C? (1)



1.2 Gravitational (force)

1.3 No. There is friction/non-conservative force (doing work)/It is not an isolated system.

1.4 Option 1

$$PE = mgh$$

= (1,8 kg)(9,8 m/s²) (1,5 m)
= 26,46 J

Option 2

$$W_W = -\Delta PE$$
(1,8 kg)(9,8 m/s²) (h - 0)cos180° = -(PE_A - PE_{ground})
(1,8 kg)(9,8 m/s²)(1,5 m)(-1) = -PE_A
PE = 26,46 J



Option 3

 $W = F\Delta x \cos\theta$ = mg\[Delta h cos\] = (1,8)(9,8) (1,5)cos\]0^o = 26,46 J



5.5 Option 1 $W_{nc} = \Delta K + \Delta U$ $W_f = \frac{1}{2}m(v_f^2 - v_0^2) + mg(h_f - h_0)$ $= \frac{1}{2}(1.8 \text{kg})(4^2 - 0.95^2)(m/s)^2 + (0 - 26.46)$ = -12,87 J

Work Done by a Variable Force

Constant Force

$$W = (F\cos\theta)s$$



(a)

Variable Force

$$W \approx (F\cos\theta)_1 \Delta s_1 + (F\cos\theta)_2 \Delta s_2 + \cdots$$

